2007

PHYSICS

1. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is
   (a) 0.5 s
   (b) 0.75 s
   (c) 0.125 s
   (d) 0.25 s

2. In an AC circuit, the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin (\omega t - \pi/2)$. The power consumption in the circuit is given by
   (a) $P = E_0 I_0 / \sqrt{2}$
   (b) $P = $ zero
   (c) $P = E_0 I_0 / 2$
   (d) $P = \sqrt{2} E_0 I_0$

3. An electric charge $10^{-3} \mu C$ is placed at the origin (0, 0) of xy-coordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be
   (a) 9 V
   (b) zero
   (c) 2 V
   (d) 4.5 V

4. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be
   (a) 1
   (b) 2
   (c) 1/4
   (d) 1/2

5. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. After 2 s, the connection is made, the current flowing (in ampere) in the circuit is
   (a) $(1 - e)$
   (b) e
   (c) e^{-1}
   (d) $(1 - e^{-1})$

6. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and 2a is
   (a) $\frac{1}{4}$
   (b) 4
   (c) 1
   (d) $\frac{1}{2}$

7. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then,
   (a) the magnetic field is zero only on the axis of the pipe.
   (b) the magnetic field is different at different points inside the page
   (c) the magnetic field at any point inside the pipe is zero
   (d) the magnetic field at all points inside the pipe is the same, but not zero

8. If M_O is the mass of an oxygen isotope $_8O^{17}$, M_p and M_n are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is
   (a) $(M_O - 8M_p)c^2$
   (b) $(M_O - 8M_p - 9M_n)c^2$
   (c) $M_O c^2$
   (d) $(M_O - 17M_n)c^2$

9. In gamma ray emission from a nucleus
   (a) both the neutron number and the proton number change
   (b) there is no change in the proton number and the neutron number
   (c) only the neutron number changes
   (d) only the proton number changes

10. If in a p-n junction diode, a square input signal of 10 V is applied as shown
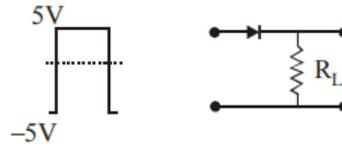

Then, the output signal across R_L will be

(a)

(b)

(c)

(d)
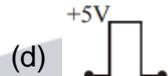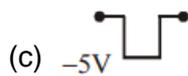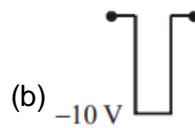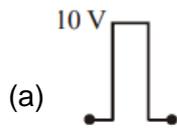
11. Photon of frequency v has a momentum associated with it. If c is the velocity of light, the momentum is
    (a) v/c (b) hvc
    (c) hv/c^2 (d) hv/c

12. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time $(t = 1)$ is
    (a) $v_0 + 2g + 3f$ (b) $v_0 + g/2 + f/3$
    (c) $v_0 + g + f$ (d) $v_0 + g/2 + f$

13. For the given uniform square lamina ABCD, whose centre is O
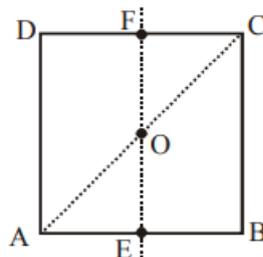
(a) $\sqrt{2}\, I_{AC} = L_{EF}$ (b) $I_{AD} = 4I_{EF}$
(c) $I_{AC} = I_{EF}$ (d) $I_{AC} = \sqrt{2}\, I_{EF}$

14. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A \cos(\omega t + \delta)$, then
    (a) $A = x_0$, $\delta = -\pi/4$ (b) $A = x_0\omega^2$, $\delta = \pi/4$
    (c) $A = x_0\omega^2$, $\delta = -\pi/4$ (d) $A = x_0\omega^2$, $\delta = 3\pi/4$

15. Charges are placed on the vertices of a square as shown. Let E be the electric field and V be the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then
(a) E remains unchanged, V changes
(b) both E and V change
(c) E and V remain unchanged
(d) E changes, V remains unchanged

16. The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially, they have the same number of atoms. Then,
(a) X will decay faster than Y
(b) Y will decay faster than X
(c) Y and X have same decay rate initially
(d) X and Y decay at same rate always

17. A Carnot engine, having an efficiency of η = 1/10 as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is
(a) 99 J   (b) 90 J
(c) 1 J   (d) 100 J

18. Carbon, silicon and germanium have four valence electrons each. At room temperature, which one of the following statements is most appropriate?
(a) The number of free conduction electron is significant in C but small in Si and Ge
(b) The number of free conduction electrons is negligibly small in all the three
(c) The number of free electrons for conduction is significant in all the three
(d) The number of free electrons for conduction is significant only in Si and Ge but small in C

19. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields E and B with a velocity v perpendicular to both E and B and comes out without any change in magnitude or direction of v. then,
(a) $v = E \times B/B^2$ (b) $v = B \times E/B^2$
(c) $v = E \times B/E^2$ (d) $v = B \times E/E^2$

20. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by $V(x) = 20 / (x^2 - 4)$ volt. The electric field E at x = 4 μm is given by
(a) $\frac{5}{3}$ V/μm and in the –ve x direction   (b) $\frac{5}{3}$ V/μm and in the +ve x direction
(c) $\frac{10}{9}$ V/μm and in the –ve x direction   (d) $\frac{10}{9}$ V/μm and in the +ve x direction

21. Which of the following transitions in hydrogen atoms emit photons of highest frequency?
(a) n = 2 to n = 6   (b) n = 6 to n = 2
(c) n = 2 to n = 1   (d) n = 1 to n = 2

22. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then, a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m.

(a) $\frac{mF}{M}$

(b) $\frac{(M+m)F}{m}$

(c) $\frac{mF}{(m+M)}$

(d) $\frac{MF}{(m+M)}$

23. Two lenses of power -15D and +5D are in contact with each other. The focal length of the combination is

(a) -20 cm

(b) -10 cm

(c) +20 cm

(d) +10 cm

24. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities K_1 and K_2, respectively. The temperature at the interface of the two sections is
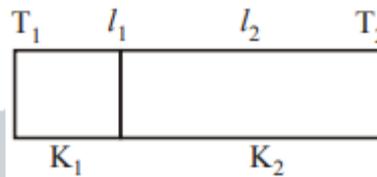
(a) $(K_2 l_2 T_1 + K_1 l_1 T_2)/(K_1 l_1 + K_2 l_2)$

(b) $(K_2 l_1 T_1 + K_1 l_2 T_2)/(K_2 l_1 + K_1 l_2)$

(c) $(K_1 l_2 T_1 + K_2 l_1 T_2)/(K_1 l_2 + K_2 l_1)$

(d) $(K_1 l_1 T_1 + K_2 l_2 T_2)/(K_1 l_1 + K_2 l_2)$

25. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of

(a) 1000

(b) 10000

(c) 10

(d) 100

26. If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(a) $C_p - C_v = R/28$

(b) $C_p - C_v = R/14$

(c) $C_p - C_v = R$

(d) $C_p - C_v = 28 R$

27. A charged particle moves through a magnetic field perpendicular to its direction. Then,
   (a) the momentum changes but the kinetic energy is constant
   (b) both momentum and kinetic energy of the particle are not constant
   (c) both momentum and kinetic energy of the particle are constant
   (d) kinetic energy changes but the momentum is constant

28. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by

(a) $\mu_0/2\pi \, (I_1 + I_2/d)^{1/2}$

(b) $\mu_0/2\pi d \, (I^2_1 + I^2_2/d)^{1/2}$

(c) $\mu_0/2\pi d \, (I_1 + I_2)$

(d) $\mu_0/2\pi d \, (I^2_1 + I^2_2)$

29. The resistance of a wire is 5 Ω at 50˚C and 6 Ω at 100˚C. The resistance of the wire at 0˚C will be
(a) 2 Ω   (b) 1 Ω
(c) 4 Ω   (d) 3 Ω

30. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is
(a) $\frac{1}{2}(K - 1)CV^2$ (b) $CV^2(K - 1)/K$
(c) $(K - 1)CV^2$ (d) zero

CHEMISTRY

31. The energies of activation for forward and reverse reactions for $A_2 + B_2 \rightleftharpoons 2\,AB$ are 180 kJ mol^{-1} and 200 kJ mol^{-1} respectively. The presence of a catalyst lowers the activation energy of both (forward and reverse) reactions by 100 kJ mol^{-1}. The enthalpy change of the reaction ($A_2 + B_2 \rightarrow 2\,AB$) in the presence of catalyst will be (in kJ mol^{-1})
    (a) 300
    (b) 120
    (c) 280
    (d) -20

32. The cell, $Zn|Zn^{2+}(1M||Cu^{2+}$ ($1M|Cu(E^\circ_{cell}$ = 1.10 V), was allowed to be completely discharged at 298 K. The relative concentration of Zn^{2+} to Cu^{2+} ($[Zn^{2+}]/[Cu^{2+}]$) is
    (a) antilog (24.08)
    (b) 37.3
    (c) $10^{37.3}$
    (d) 9.65×10^4

33. The pK_a of a weak acid (HA) is 4.5. The pOH of an aqueous buffered solution of HA in which 50% of the acid ionised is
    (a) 4.5
    (b) 2.5
    (c) 9.5
    (d) 7.0

34. Consider the reaction, $2A + B \rightarrow$ products
    When concentration of B alone was doubled, the half-life did not change. When the concentration of A alone was doubled, the rate increased by two times. The unit of rate constant for this reaction is
    (a) L mol^{-1}s^{-1}
    (b) no unit
    (c) mol L^{-1}s^{-1}
    (d) s^{-1}

35. Identify the incorrect statement among the following.
    (a) d-block elements show irregular and erratic chemical properties among themselves
    (b) La and Lu have partially filled d orbitals and no other partially filled orbitals
    (c) The chemistry of various lanthanoids is very similar
    (d) 4f and 5f orbitals are equally shielded

36. Which one of the following has a square planar geometry?
    (a) $[CoCl_4]^{2-}$
    (b) $[FeCl_4]^{2-}$
    (c) $[NiCl_4]^{2-}$
    (d) $[PtCl_4]^{2-}$

37. Which of the following molecules is expected to rotate the plane of plane-polarised light?
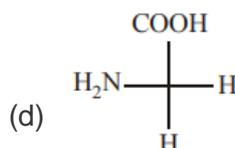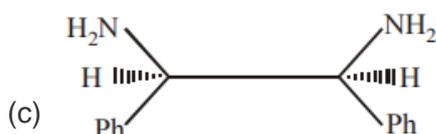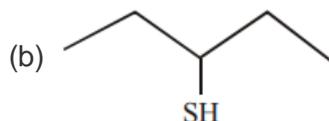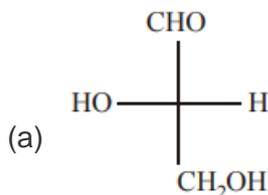
38. The secondary structure of a protein refers to
    (a) a-helical backbone
    (b) hydrophobic interactions
    (c) sequence of a-amino acids
    (d) fixed configuration of the polypeptide backbone

39. Which of the following reaction will yield, 2, 2-dibromopropane?
    (a) $CH_3–C≡CH + 2HBr →$ (b) $CH_3CH=CHBr + HBr →$
    (c) $CH≡CH + 2HBr →$ (d) $CH_3 – CH==CH_2 + HBr →$

40. In the chemical reaction, $CH_3CH_2NH_2 + CHCl_3 + 3KOH →(A) + (B) + 3H_2O$, the compounds (A) and (B) are respectively
    (a) C_2H_5CN and $3KCl$ (b) $CH_3CH_2CONH_2$ and $3KCl$
    (c) C_2H_5NC and K_2CO_3 (d) C_2H_5NC and $3KCl$

41. The reaction of toluene with Cl_2 in the presence of $FeCl_3$ gives predominantly
    (a) benzoyl chloride   (b) benzyl chloride
    (c) o – and p-chlorotoluene   (d) m-chlorotoluene

42. Presence of a nitro group in a benzene ring
    (a) activates the ring towards electrophilic substitution
    (b) renders the ring basic
    (c) deactivates the ring towards nucleophilic substitution
    (d) deactivates the ring towards electrophilic substitution

43. In which of the following ionisation processes, the bond order has increased and the magnetic behaviour has changed?
    (a) $C_2 → C^+_2$ (b) $NO → NO^+$
    (c) $O_2 → O^+_2$ (d) $N_2 → N^+_2$

44. The actinoids exhibit more number of oxidation states in general than the lanthanoids. This is because
    (a) the 5f orbitals are more buried than the 4f orbitals
    (b) there is a similarity between 4f and 5f orbitals in their angular part of the wave function
    (c) the actinoids are more reactive than the lanthanoids
    (d) the 5f orbitals extend further from the nucleus than the 4f orbitals

45. Equal masses of methane and oxygen are mixed in an empty container at 25˚C. The fraction of the total pressure exerted by oxygen is
    (a) $\frac{2}{3}$ (b) $\frac{1}{3} \times \frac{273}{298}$
    (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

46. A 5.25% solution of a substance is isotonic with a 1.5% solution of urea (molar mass = 60 g mol^{-1}) in the same solvent. If the densities of both the solution are assumed to be equal to 1.0 g cm^{-3}, the molar mass of the substance will be
(a) 90.0 g mol^{-1}
(b) 115.0 g mol^{-1}
(c) 105.0 g mol^{-1}
(d) 210.0 g mol^{-1}

47. Assuming that water vapour is an ideal gas, the internal energy change (ΔU) when 1 mole of water is vapourised at 1 bar pressure and 100°C, (Given : molar enthalpy of vaporisation of water at 1 bar and 373 K = 41 kJ mol^{-1} and R = 8.3 J mol^{-1}K^{-1}) will be
(a) 4.100 kJ mol^{-1}
(b) 3.7904 kJ mol^{-1}
(c) 37.904 kJ mol^{-1}
(d) 41.00 kJ mol^{-1}

48. In a saturated solution of the sparingly soluble strong electrolyte AgIO$_3$ (Molecular mass = 283), the equilibrium which sets in, is AgIO$_3$(s) $\rightleftharpoons$ Ag$^+$(aq) + IO$_3^-$(aq) if the solubility product constant, K$_{sp}$ of AgIO$_3$ at a given temperature is 1.0 x 10^{-8}, what is the mass of AgIO$_3$ contained in 100 mL of its saturated solution?
(a) 28.3 x 10^{-2} g
(b) 2.83 x 10^{-3} g
(c) 1.0 x 10^{-7} g
(d) 1.0 x 10^{-4} g

49. A radioactive element gets spilled over the floor of a room. Its half-life period is 30 days. If the initial activity is ten times the permissible value, after how many days will it be safe to enter the room?
(a) 1000 days
(b) 300 days
(c) 10 days
(d) 100 days

50. Which one of the following conformations of cyclohexane is chiral?
(a) Twist boat
(b) Rigid
(c) Chair
(d) Boat

51. Which of the following is the correct order of decreasing S$_N$2 reactivity?
(a) RCH$_2$X > R$_3$CX > R$_2$CH X
(b) RCH$_2$X > R$_2$CHX > R$_3$C X
(c) R$_3$C X > R$_2$CHX > RCH$_2$ X
(d) R$_2$CHX > R$_3$CX > RCH$_2$ X

52. In the following sequence of reactions

$$CH3CH2OH \xrightarrow{P+I_2} A \xrightarrow[\text{ether}]{Mg} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$$
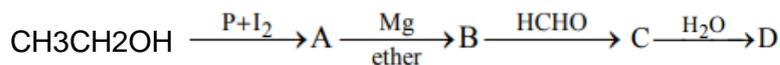
the compound 'D' is
(a) butanal
(b) n-butyl alcohol
(c) n-propyl alcohol
(d) propanal

53. Which of the following sets of quantum numbers represents the highest energy of an atom?
(a) n = 3, l = 1, m = 1, s = + 1/2
(b) n = 3, l = 2, m = 1, s = + 1/2
(c) n = 4, l = 0, m = 0, s = + 1/2
(d) n = 3, l = 0, m = 0, s = + 1/2

54. Which of the following hydrogen bonds is the strongest?
    (a) O – H…N
    (b) F – H…F
    (c) O – H…O
    (d) O – H…F

55. In the reaction, $2Al(s) + 6HCl(aq) \rightarrow 2Al^{3+}(aq) + 6Cl^-(aq) + 3H_2(g)$
    (a) 6 L HCl (aq) is consumed for every 3 L $H_2(g)$ produced
    (b) 33.6 L H_2 (g) is produced regardless of temperature and pressure for every mole of Al that reacts
    (c) 67.2 L $H_2(g)$ at STP is produced for every mole of Al that reacts
    (d) 11.2 L $H_2(g)$ at STP is produced for every mole of HCl(aq) consumed

56. Regular use of which of the following fertilisers increases the acidity of soil?
    (a) Potassium nitrate
    (b) Urea
    (c) Superphosphate of lime
    (d) Ammonium sulphate

57. Identify the correct statement regarding a spontaneous process
    (a) For a spontaneous process in an isolated system, the change in entropy is positive
    (b) Endothermic processes are never spontaneous
    (c) Exothermic processes are always spontaneous
    (d) Lowering of energy in the reaction process is the only criterion for spontaneity

58. Which of the following nuclear reactions will generate an isotope?
    (a) Neutron particle emission
    (b) Positron emission
    (c) α-particle emission
    (d) β-particle emission

59. The equivalent conductances of two strong electrolytes at infinite dilution in H_2O (where ions move freely through a solution) at 25°C are given below

$$\Lambda^{\circ}_{CH_3COONa} = 91.0 \ S \ cm^2/equiv$$

$$\Lambda^{\circ}_{HCl} = 426.2 \ S \ cm^2/equiv$$

What additional information/quantity one needs to calculate Λ° of an aqueous solution of acetic acid?
    (a) Λ° of NaCl
    (b) Λ° of CH_3COOK
    (c) The limiting equivalent conductance of $H^+(\lambda^{\circ}_{H+})$
    (d) Λ° of chloroacetic acid ($ClCH_2COOH$)

60. Which one of the following is the strongest base in aqueous solution?
    (a) Trimethylamine
    (b) Aniline
    (c) Dimethylamine
    (d) Methylamine

MATHEMATICS

61. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then, the common ratio of this progression equals
    (a) $\frac{1}{2}(1 - \sqrt{5})$
    (b) $\frac{1}{2}\sqrt{5}$
    (c) $\sqrt{5}$
    (d) $\frac{1}{2}(\sqrt{5} - 1)$

62. If $\sin^{-1}(x/5) + \csc^{-1}(5/4) = \frac{\pi}{2}$, then a value of x is
    (a) 1
    (b) 3
    (c) 4
    (d) 5

63. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5th and 6th terms is zero, then $\frac{a}{b}$ equals
    (a) $\frac{5}{n-4}$
    (b) $\frac{6}{n-5}$
    (c) $\frac{n-5}{6}$
    (d) $\frac{n-4}{5}$

64. The set S = {1, 2, 3, …, 12} is to be partitioned into three sets. A, B, C of equal size. Thus, A ∪ B ∪ C = S, A ∩ B = B ∩ C = A ∩ C = φ. The number of ways to partition S is
    (a) $12!/3! \, (4!)^3$
    (b) $12!/3!(3!)^4$
    (c) $12!/(4!)^3$
    (d) $12!/(3!)^4$

65. The largest interval lying in (-π/2, π/2) for which the function
    $$\left[f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)\right] \text{ is defined, is}$$
    (a) [0, π]
    (b) (- π/2, π/2)
    (c) [-π/4, π/2]
    (d) [0, π/2]

66. A body weighing 13 kg is suspended by two strings 5 m and 12 m long, their other ends being fastened to the extremities of a rod 13 m long. If the rod be so held that the body hangs immediately below the middle point. The tensions in the strings are
    (a) 12 kg and 13 kg
    (b) 5 kg and 5 kg
    (c) 5 kg and 12 kg
    (d) 5 kg and 13 kg

67. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is
    (a) 1/729
    (b) 8/9
    (c) 8/729
    (d) 8/243

68. Consider a family of circles which are passing through the point (-1, 1) and are touching X-axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval
    (a) 0 < k < 1/2
    (b) k ≥ 1/2
    (c) -1/2 ≤ k ≤ 1/2
    (d) k ≤ 1/2

69. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then cos α equals
   (a) $1/\sqrt{3}$
   (b) $1/2$
   (c) 1
   (d) $1/\sqrt{2}$

70. The differential equation of all circles passing through the origin and having their centres on the X-axis is
   (a) $x^2 = y^2 + xy \frac{dy}{dx}$
   (b) $x^2 = y^2 + 3xy \frac{dy}{dx}$
   (c) $y^2 = x^2 + 2xy \frac{dy}{dx}$
   (d) $y^2 = x^2 - 2xy \frac{dy}{dx}$

71. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of (p + q) is
   (a) 2
   (b) $\frac{1}{2}$
   (c) $\frac{1}{\sqrt{2}}$
   (d) $\sqrt{2}$

72. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (=a) subtends an angle of 60˚ at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30˚. The height of the tower is
   (a) $2a/\sqrt{3}$
   (b) $2a\sqrt{3}$
   (c) $a/\sqrt{3}$
   (d) $a\sqrt{3}$

73. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots + {}^{20}C_{10}$ is
   (a) $- {}^{20}C_{10}$
   (b) $\frac{1}{2} {}^{20}C_{10}$
   (c) 0
   (d) ${}^{20}C_{10}$

74. The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a
   (a) ellipse
   (b) parabola
   (c) circle
   (d) hyperbola

75. If $| z + 4| \leq 3$, then the maximum value of $|z + 1|$ is
   (a) 4
   (b) 10
   (c) 6
   (d) 0

76. The resultant of two forces P N and 3 N is a force of 7 N. If the direction of the 3 N force were reversed, the resultant would be $\sqrt{19}$ N. The value of P is
   (a) 5N
   (b) 6 N
   (c) 3 N
   (d) 4 N

77. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is
   (a) 0.06
   (b) 0.14
   (c) 0.2
   (d) 0.7

78. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for x ≠ 0, y ≠ 0, then D is

    (a) divisible by neither x nor y   (b) divisible by both x and y

    (c) divisible by x but not y   (d) divisible by y but not x

79. For the hyperbola $x^2/\cos^2 \alpha - y^2/\sin^2 \alpha = 1$, which of the following remains constant when α varies?

    (a) Eccentricity   (b) Directrix

    (c) Abscissae of vertices   (d) Abscissae of foci

80. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of X-axis and Y-axis, then the angle that the line makes with the positive direction of the Z-axis is

    (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

    (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

81. A value of C for which the conclusion of Mean Value Theorem holds for the function f(x) = $\log_e x$ on the interval [1, 3] is

    (a) $2\log_3 e$ (b) $\frac{1}{2}\log_e 3$

    (c) $\log_3 e$ (d) $\log_e 3$

82. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

    (a) (π/4, π/2)   (b) (-π/2, π/4)

    (c) (0, π/2)   (d) (-π/2, π/2)

83. Let A = $\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$.

    If $|A^2| = 25$, then |α| equals

    (a) 5^2 (b) 1

    (c) $\frac{1}{5}$ (d) 5

84. The sum of the series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots$ upto infinity is

    (a) e^{-2} (b) e^{-1}

    (c) $e^{-1/2}$ (d) $e^{+1/2}$

85. If u an v are unit vectors and θ is the acute angle between them, then 2u x 3v is unit vector for

    (a) exactly two values of θ   (b) more than two values of θ

    (c) no value of θ   (d) exactly one value of θ

86. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is

    (a) $\tan^{-1}\frac{b}{ac}$ (b) 45°

    (c) $\tan^{-1}\frac{bc}{a(c-a)}$ (d) $\tan^{-1}\frac{bc}{a}$

87. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is
(a) 40
(b) 20
(c) 80
(d) 60

88. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is
(a) (-1, 1)
(b) (0, 2)
(c) (2, 4)
(d) (-2, 0)

89. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are
(a) (4, 9, -3)
(b) (4, -3, 3)
(c) (4, 3, 5)
(d) (4, 3, -3)

90. Let $a = i + j + k$, $b = i - j + 2k$ and $c = xi + (x - 2)j - k$. If the vector c lies in the plane of a and b, then x equals
(a) 0
(b) 1
(c) -4
(d) -2