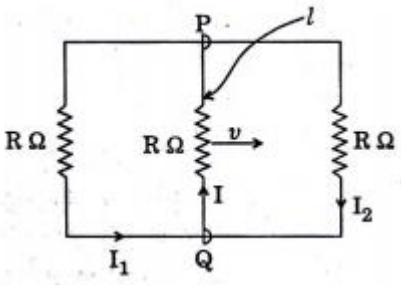
2010

PHYSICS

1. A rectangular loop has a sliding connector PQ of length l and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(a) $I_1 = -I_2 = \frac{Blv}{R}$, $I = \frac{2Blv}{R}$
(b) $I_1 = I_2 = \frac{Blv}{3R}$, $I = \frac{2Blv}{3R}$
(c) $I_1 = I_2 = I = \frac{Blv}{R}$
(d) $I_1 = I_2 = \frac{Blv}{6R}$, $I = \frac{Blv}{3R}$

2. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then, the ratio t_1 / t_2 will be

(a) 1
(b) $\frac{1}{2}$
(c) $\frac{1}{4}$
(d) 2

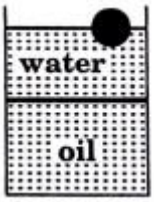
Directions Q. Nos. 3-4 contain Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

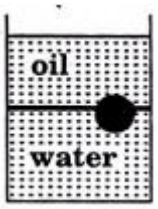
(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is false

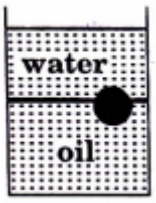
3. **Statement I** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.
Statement II Principle of conservation of momentum holds true for all kinds of collisions.

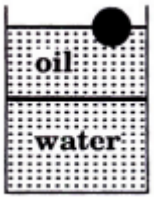
4. **Statement I** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.
Statement II Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

5. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?

(a) 

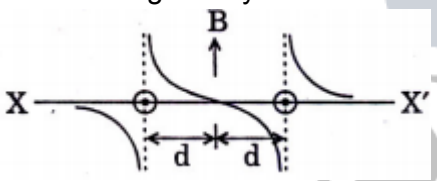
(b) 

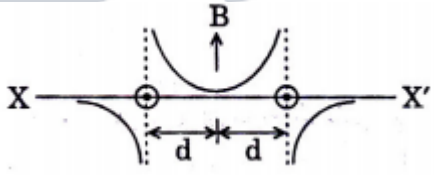
(b) 

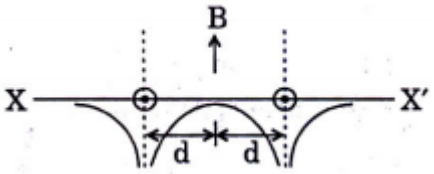
(d) 

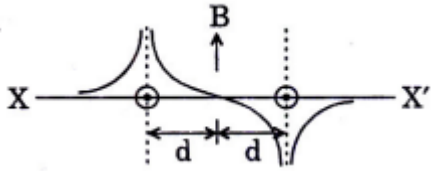
6. A particle is moving with velocity v = k(y I + xj), where k is a constant. The general equation for its path is
(a) $y = x^2$ + constant
(b) $y^2 = x$ + constant
(c) xy = constant
(d) $y^2 = x^2$ + constant

7. Two long parallel wires are at a distance 2d apart. They carry steady equal current flowing out of the plane of the paper as shown. The variation of the magnetic field along the line XX' is given by

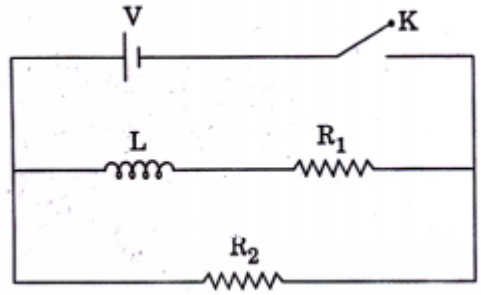
(a) 

(b) 

(c) 

(d) 

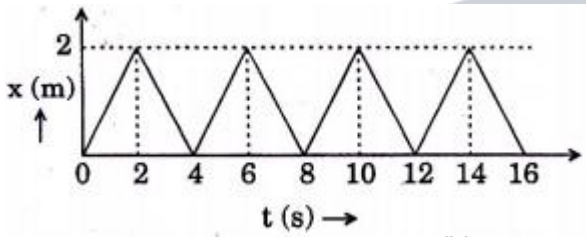
8. In the circuit shown below, the key K is closed at t = 0. The current through the battery is



(a) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞ (b) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1+R_2)}{R_1R_2}$ at t = ∞

(c) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = ∞ (d) $\frac{V(R_1+R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

9. The figure shows the position-time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(a) 0.4 Ns (b) 0.8 Ns
(c) 1.6 Ns (d) 0.2 Ns

Directions Q. Nos. 10-11 are based on the following paragraph.
A nucleus of mass M + Δm is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

10. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then
(a) $E_2 = 2E_1$ (b) $E_1 > E_2$
(c) $E_2 > E_1$ (d) $E_1 = 2E_2$

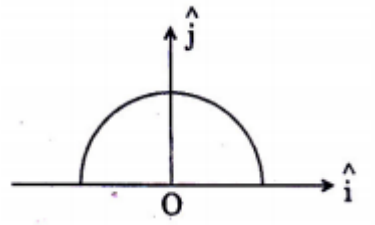
11. The speed of daughter nuclei is
(a) $c\frac{\Delta m}{M+\Delta m}$ (b) $c\sqrt{\frac{2\Delta m}{M}}$
(c) $c\sqrt{\frac{\Delta m}{M}}$ (d) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$

12. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(a) $\frac{A-Z-8}{Z-4}$ (b) $\frac{A-Z-4}{Z-8}$

(c) $\frac{A-Z-12}{Z-4}$ (d) $\frac{A-Z-4}{Z-2}$

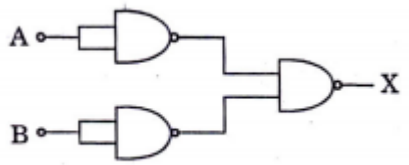
13. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field E at the centre O is



(a) $(q/4\pi^2\varepsilon_0 r^2)j$ (b) $(-q/4\pi^2\varepsilon_0 r^2)\,j$

(c) $(-q/2\pi^2\varepsilon_0 r^2)\,j$ (d) $(-q/2\pi^2\varepsilon_0 r^2)\,j$

14. The combination of gates shown below yields



(a) OR gate (b) NOT gate

(c) XOR gate (d) NAND gate

15. A diatomic ideal gas is used in a car engine as the working substance. If during the adiabatic expansion part during the adiabatic expansion part of the cycle, volume of the gas increases from V to 32 V, the efficiency of the engine is

(a) 0.5 (b) 0.75

(c) 0.99 (d) 0.25

16. If a source of power 4 kW produces 10^{20} photons/second, the radiation belong to a part of the spectrum called

(a) X-rays (b) 5, 1, 5

(c) 5, 5, 2 (d) 4, 4, 2

17. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(a) 5, 1, 2 (b) 5, 1, 5

(c) 5, 5, 2 (d) 4, 4, 2

18. In a series L-C-R circuit, R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the L-C-R circuit is
(a) 305 W (b) 210 W
(c) zero (d) 242 W

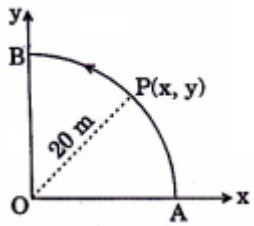
19. Let three be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0 (5/4 - r/R)$ upto r = R, and $\rho(r) = 0$ for r > R, where r is the distance from the origin. The electric field at a distance r(r < R) from the origin is given by
(a) $4\pi\rho_0 r/3\varepsilon_0 (5/3 - r/R)$ (b) $\rho_0 r/4\varepsilon_0 (5/3 - r/R)$
(c) $4\rho_0 r/3\varepsilon_0 (5/4 - r/R)$ (d) $\rho_0 r/3\varepsilon_0 (5/4 - r/R)$

20. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = a/x^{12} - b/x^6$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is
(a) $b^2/2a$ (b) $b^2/12a$
(c) $b^2/4a$ (d) $b^2/6a$

21. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 16 g cm^{-3}, the dielectric constant of the liquid is
(a) 4 (b) 3
(c) 2 (d) 1

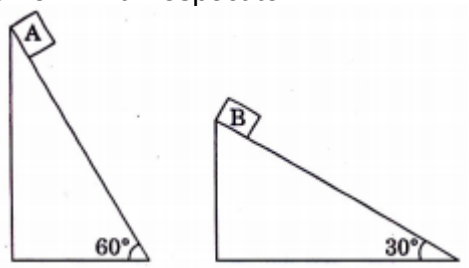
22. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly
(a) $\alpha_1 + \alpha_2/2$, $\alpha_1 + \alpha_2$ (b) $\alpha_1 + \alpha_2$, $\alpha_1 + \alpha_2/2$
(c) $\alpha_1 + \alpha_2$, $\alpha_1\alpha_2/\ \alpha_1 + \alpha_2$ (d) $\alpha_1 + \alpha_2/2$, $\alpha_1 + \alpha_2/2$

23. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metre and t is in second. The radius of the path is 20 m. the acceleration of P when t = 2 s is nearly



(a) 13 ms^{-2} (b) 12 ms^{-2}
(c) 7.2 ms^{-2} (d) 14 ms^{-2}

24. Two fixed frictionless inclined plane making an angle 30° and 60° with the vertical are shown in the figure. Two block A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



(a) 4.9 ms^{-2} in horizontal
(b) 9.8 ms^{-2} in vertical direction
(c) Zero
(d) 4.9 ms^{-2} in vertical direction

25. For a particle in uniform circular motion the acceleration a at a point P(R,θ) on the circle of radius R is (here θ is measured from the X-axis)
(a) $-v^2/R \cos\theta\, i + v^2/R \sin\theta\, j$
(b) $-v^2/R \sin\theta\, i + v^2/R \cos\theta\, j$
(c) $-v^2/R \cos\theta\, i - v^2/R \sin\theta\, j$
(d) $v^2/R\, I + v^2/R\, j$

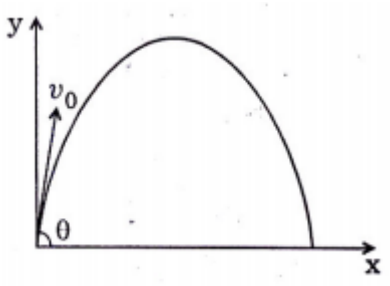
Direction Q. Nos. 26-28 are based on the following paragraph.
A initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

26. As the beam enters the medium, it will
(a) diverge
(b) converge
(c) diverge near the axis and converge near the periphery
(d) travel as a cylindrical beam

27. The initial shape of the wavefront of the beam is
(a) convex
(b) concave
(c) convex near the axis and concave near the periphery
(d) planar

28. The speed of light in the medium is
(a) minimum on the axis of the beam
(b) the same everywhere in the beam
(c) directly proportional to the intensity I
(d) maximum on the axis of the beam

29. A small particle of mass m is projected at an angle θ with the X-axis with an initial velocity v_0 in the xy-plane as shown in the figure. At a time $t < v_0 \sin\theta / g$, the angular momentum of the particle is



(a) $-mgv_0t^2 \cos\theta\, j$
(b) $mgv_0t \cos\theta$
(c) $-\frac{1}{2} mgv_0t^2 \cos\theta\, k$
(d) $\frac{1}{2} mgv_0t^2 \cos\theta\, i$

30. The equation of a wave on a string of linear mass density of 0.04 kg m^{-1} is given by y = 0.02 (m) sin [2π(t/0.04(s) – x/0.50(m))].
The tension in the string is
(a) 4.0 N
(b) 12.5 N
(c) 0.5 N
(d) 6.25 N

CHEMISTRY

31. The standard enthalpy of formation of NH_3 is -46.0 kJ mol^{-1} If the enthalpy of formation of H_2 from its atoms is -436 kJ mol^{-1} and that of N_2 is -712 kJ mol^{-1}, the average bond enthalpy of N-H bond in NH_3 is
    (a) -964 kJ mol^{-1} (b) +352 kJ mol^{-1}
    (c) + 1056 kJ mol^{-1} (d) -1102 kJ mol^{-1}

32. The time for half-life period of a certain reaction, A → products is 1 h. When the initial concentration of the reactant 'A', is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol^{-1}, if it is a zero order reaction?
    (a) 4 h (b) 0.5 h
    (c) 0.25 h (d) 1 h

33. A solution containing 2.675 g of $CoCl_3 . 6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is
    (At mass of Ag = 108 u)
    (a) $[Co(NH_3)_6]Cl_3$ (b) $[CoCl_2(NH_3)_4]Cl$
    (c) $[CoCl_3(NH_3)_3]$ (d) $[CoCl(NH_3)_5]Cl_2$

34. Consider the reaction, Cl_2 (aq) + H_2S(aq) → S(s) + $2H^+$(aq) + $2Cl^-$(aq)
    The rate equation for this reaction is, rate = k[Cl_2] [H_2S]
    Which of these mechanisms is/are consistent with this rate equation?
    I. $Cl_2 + H_2S \rightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)
    $Cl^+ + HS^- \rightarrow H^+ + Cl^- + S$ (fast)
    II. $H_2S \Leftrightarrow H^+ HS^-$ (fast, equilibrium)
    $Cl_2 + HS^- \rightarrow 2Cl^- + H^+ + S$ (slow)
    (a) (II) only (b) Both (I) and (II)
    (c) Neither (I) nor (II) (d) (I) only

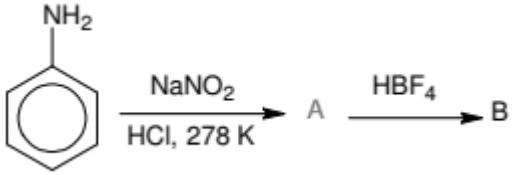
35. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established?
    (Given, vapour pressure of H_2O at 300 K is 3170 Pa; R = 8.314 JK^{-1} mol^{-1})
    (a) 5.56 x 10^{-3} mol (b) 1.53 x 10^{-2} mol
    (c) 4.46 x 10^{-2} mol (d) 1.27 x 10^{-3} mol

36. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is
    (a) propene (b) 1-butene
    (c) 2-butene (d) ethane

37. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mole of sodium sulphate is dissolved in 1 kg of water is (K_f = 1.86 K kg mol^{-1})
    (a) 0.0372 K (b) 0.0558 K
    (c) 0.0744 K (d) 0.0186 K

38. From amongst the following alcohols, the one that would react fastest with conc. HCl and anhydrous $ZnCl_2$, is
(a) 2-butanol
(b) 2-methylpropan-2-ol
(c) 2-methylpropanol
(d) 1-butanol

39. In the chemical reactions,



The compounds 'A' and 'B' respectively are
(a) nitrobenzene and fluorobenzene
(b) phenol and benzene
(c) benzene diazonium chloride and fluorobenzene
(d) nitrobenzene and chlorobenzene

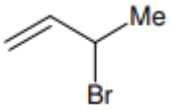
40. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralisation. The percentage of nitrogen in the compound is
(a) 59.0
(b) 47.4
(c) 23.7
(d) 29.5

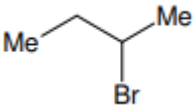
41. The energy required to break one mole of Cl-Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl-Cl bond is
(c = 3t x 10^8 ms^{-1} and N_A = 6.02 X 10^{23} mol^{-1})
(a) 94 nm
(b) 640 nm
(c) 700 nm
(d) 494 nm

42. Ionisation energy of He^+ is 19.6 x 10^{-18} J $atom^{-1}$. The energy of the first stationary state (n=1) of Li^{2+} is
(a) 4.41 x 10^{-16} J $atom^{-1}$
(b) -4.41 x 10^{-17} J $atom^{-1}$
(c) -2.2 x 10^{-15} J $atom^{-1}$
(d) 8.82 x 10^{-17} J $atom^{-1}$

43. Consider the following bromides

(I) 

(II) 

(III) 

The correct order of S_N1 reactivity is

(a) (II) > (III) > (I)
(b) (II) > (I) > (III)
(c) (III) > (II) > (I)
(d) (I) > (II) > (III)

44. Which one of the following has an optical isomer?

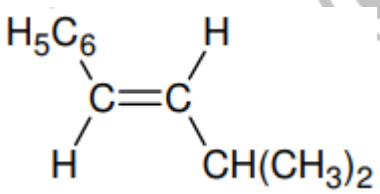
(a) $[Zn(en)(NH_3)_2]^{2+}$
(b) $[Co(en)_3]^{3+}$
(c) $[Co(H_2O)_4(en)]^{3+}$
(d) $[Zn(en)_2]^{2+}$

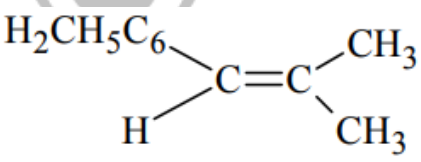
45. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressure of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1}).

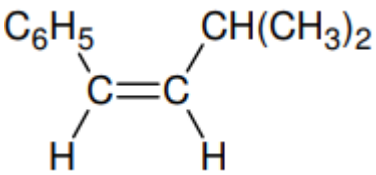
(a) 72.0 kPa
(b) 36.1 kPa
(c) 96.2 kPa
(d) 144.5 kPa

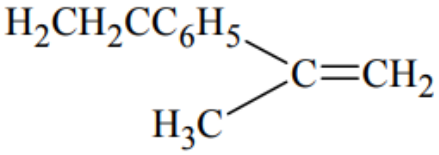
46. The main product of the following reaction is $C_6H_5CH_2CH(OH)CH(CH_3)_2$

$\xrightarrow{\text{conc. } H_2SO_4}$?

(a) 

(b) 

(c) 

(d) 

47. Three reactions involving $H_2PO^-_4$ are given below

I. $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO^-_4$
II. $H_2PO^-_4 + H_2O \rightarrow HPO^{2-}_4 + H_3O^+$
III. $H_2PO^-_4 + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2PO^-_4$ act as an acid?

(a) (II) only
(b) (I) and (II)
(c) (III) only
(d) (I) only

48. In aqueous solution, the ionisation constants for carbonic acid are
$K_1 = 4.2 \times 10^{-7}$ and $K_2 = 4.8 \times 10^{-11}$
Select the correct statement for a saturated 0.034 M solution of the carbonic acid.
(a) The concentration of CO^{2-}_3 is 0.034 M
(b) The concentration of CO^{2-}_3 is greater than that of HCO^-_3
(c) The concentration of H^+ and HCO^-_3 are approximately equal
(d) The concentration of H^+ is double that of CO^{2-}_3

49. The edge length of a face centred cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is
(a) 288 pm (b) 398 pm
(c) 618 pm (d) 144 pm

50. The correct order of increasing basicity of the given conjugate bases (R = CH_3) is
(a) RCOO' < HC ═ C' < R' < N'H_2 (b) R' < HC ≡ C' < RCOO' < N'H_2
(c) RCOO' < N'H_2 < HC ≡ C' < R (d) RCOO' < HC ≡ C' < N'H_2 < R'

51. The correct sequence which shows decreasing order of the ionic radii of the elements is
(a) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$ (b) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$
(c) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$ (d) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

52. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 L of 0.05 M solution of silver nitrate to start the precipitation of AgBr is
(a) 1.2×10^{-10} g (b) 1.2×10^{-9} g
(c) 6.2×10^{-5} g (d) 5.0×10^{-8} g

53. The Gibbs energy for the decomposition of Al_2O_3 at 500°C is as follows $\frac{2}{3}$ Al_2O_3 → $\frac{4}{3}$ Al + O_2, Δ_rG = +966 kJ mol^{-1}
The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is atleast
(a) 4.5 V (b) 3.0 V
(c) 2.5 V (d) 5.0 V

54. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions?
(a) 9 (b) 10
(c) 11 (d) 8

55. Percentage of free space in cubic close packed structure and in body centred packed structure are respectively
(a) 30% and 26% (b) 26% and 32%
(c) 32% and 48% (d) 48% and 26%

56. Out of the following, the alkene that exhibits optical isomerism is
(a) 3-methyl-2-pentene (b) 4-methyl-1-pentene
(c) 3-methyl-1-pentene (d) 2-methyl-2-pentene

57. Biuret test is not given by
    (a) carbohydrates (b) polypeptides
    (c) urea (d) proteins

58. The correct order of $E^{\circ}_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is
    (a) Mn > Cr > Fe > Co (b) Cr > Fe > Mn > Co
    (c) Fe > Mn > Cr > Co (d) Cr > Mn > Fe > Co

59. The polymer containing strong intermolecular forces e.g., hydrogen bonding, is
    (a) Teflon (b) nylon-66
    (c) polystyrene (d) natural rubber

60. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when
    (a) $T_e > T$ (b) $T > T_e$
    (c) T_e is 5 times T (d) $T = T_e$

MATHEMATICS

61. Consider the following relations
R = {(x, y) | x, y are real numbers w};
x = wy for some rational number w};
S = {(m/n, p/q)| m, n, p and q are integers such that n, q ≠ 0 and qm = pn}. Then,
(a) R is an equivalence relation but S is not an equivalence relation
(b) Neither R nor S is an equivalence relation
(c) S is an equivalence relation but R is not an equivalence relation
(d) R and S both are equivalence relation

62. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals
(a) 0 (b) 1
(c) 2 (d) ∞

63. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009}$ is equal to
(a) -2 (b) -1
(c) 1 (d) 2

64. Consider the system of linear equations
$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$
The system has
(a) infinite number of solutions (b) exactly 3 solutions
(c) a unique solution (d) no solution

65. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn, two balls are taken out at random and then transferred to the other. The number of ways in which this can be done, is
(a) 3 (b) 36
(c) 66 (d) 108

66. If f : (-1, 1) → R is a differentiable function with f(0) = -1 and f'(0) = 1. Let $g(x) = [f(2f(x) + 2)]^2$. Then, g'(0) is equal to
(a) 4 (b) -4
(c) 0 (d) -2

67. Let f : R → R be a positive increasing function with $\lim_{x\to\infty} \frac{f(2X)}{f(x)} = 1$. Then, $\lim_{x\to\infty} \frac{f(2X)}{f(x)}$ is equal to
(a) 1 (b) $\frac{2}{3}$
(c) $\frac{3}{2}$ (d) 3

68. Let p(x) be a function defined on R such that $\lim_{x\to\infty} \frac{f(3X)}{f(x)}$ = 1, p'(x) = p'(1-x), for all x Ɛ [0, 1], p(0) = 1 and p(1) = 41. Then, $\int_0^1 p(x)$ dx equals
(a) $\sqrt{41}$ (b) 21
(c) 41 (d) 42

69. A person is to count 4500 currency notes. Let a_n denotes the number of notes he counts in the nth minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11} \ldots$ are in AP with common difference -2, then the time taken by him to count all notes, is
(a) 24 min (b) 34 min
(c) 125 min (d) 135 min

70. The equation of the tangent to the curve $y = x + 4/x^2$, that is parallel to the X-axis is
(a) y = 0 (b) y = 1
(c) y = 2 (d) y = 3

71. The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and x = $\frac{3\pi}{2}$ is
(a) $(4\sqrt{2} - 2)$ sq units (b) $(4\sqrt{2} + 2)$ sq units
(c) $(4\sqrt{2} - 1)$ sq units (d) $(4\sqrt{2} + 1)$ sq units

72. Solution of the differential equation cos x dy = y(sin x – y) dx, $0 < x < \frac{\pi}{2}$, is
(a) sec x = (tanx + C)y (b) ysec x = tanx + C
(c) ytan x = sec x + C (d) tan x = (sec x + C)y

73. Let a = j – k and c = I – j – k. Then, the vector b satisfying a x b + c = 0 and a.b = 3, is
(a) –i + j – 2k (b) 2i – j + 2k
(c) i – j – 2k (d) i + j – 2k

74. If the vectors a = i – j + 2 k, b = 2i + 4j + k and c = λi + j + μk are mutually orthogonal, then (λ, μ) is equal to
(a) (-3, 2) (b) (2, -3)
(c) (-2, 3) (d) (3, -2)

75. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is
(a) x = 1 (b) 2x + 1 = 0
(c) x = -1 (d) 2x – 1 = 0

76. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then, the distance between L and K is
(a) $\frac{23}{\sqrt{15}}$ (b) $\sqrt{17}$
(c) $\frac{17}{\sqrt{15}}$ (d) $\frac{23}{\sqrt{17}}$

77. A line AB in three-dimensional space makes angles 45° and 120° with the positive X-axis and the positive Y-axis, respectively. If AB makes an acute angle θ with the positive Z-axis, then θ equals
(a) 30° (b) 45°
(c) 60° (d) 75°

78. Let S be a non-empty subset of R. Consider the following statement P : There is a rational number x Ɛ S such that x > 0.
Which of the following statements is the negation of the statement P?
(a) There is a rational number x Ɛ S such that x ≤ 0
(b) There is no rational number x Ɛ S such that x ≤ 0
(c) Every rational number x Ɛ S satisfies x ≤ 0
(d) x Ɛ S and x ≤ 0 ⇒ x is not rational

79. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then, tan 2α is equal to
(a) $\frac{25}{16}$ (b) $\frac{56}{33}$
(c) $\frac{19}{12}$ (d) $\frac{20}{7}$

80. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points, if
(a) -85 < m < -35 (b) -35 < m < 15
(c) 15 < m < 65 (d) 35 < m < 85

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is
(a) $\frac{5}{2}$ (b) $\frac{11}{2}$
(c) 6 (d) $\frac{13}{2}$

82. A urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colour, is
(a) $\frac{1}{3}$ (b) $\frac{2}{7}$
(c) $\frac{1}{21}$ (d) $\frac{2}{23}$

83. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is
(a) there is a regular polygon with $\frac{r}{R} = \frac{1}{2}$
(b) there is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$
(c) there is a regular polygon with $\frac{r}{R} = \frac{2}{3}$
(d) there is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

84. The number of 3 x 3 non-singular matrices, with four entries as 1 and all other entries as 0, is
(a) less than 4 (b) 5
(c) 6 (d) atleast 7

85. Let f : R → R be defined by

$$f(x)=\begin{cases} k-2x, & \text{if } x \le -1 \\ 2x+3, & \text{if } x > -1 \end{cases}$$

If f has a local minimum at x = -1, then a possible value of k is
(a) 1 (b) 0
(c) $-\frac{1}{2}$ (d) atleast 7

Direction This section contains 5 multiple choice questions numbered 86 to 90. Each question contains Statement I (Assertion) and Statement II (Reason). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct.
(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

86. Four numbers are chosen at random (without replacement) from the set {1, 2, 3,…, 20}.
Statement I The probability that the chosen numbers when arranged in some order will form an AP, is $\frac{1}{85}$.
Statement II If the four chosen numbers from an AP, then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}.

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)^{10}C_j$, $S_2 = \sum_{j=1}^{10} j^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$
Statement I $S_3 = 55 \times 2^9$
Statement II $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$

88. **Statement I** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane x – y + z = 5.
Statement II The plane x – y + z = 5 bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

89. Let f : R → R be a continuous function defined by $f(x) = 1/e^x + 2e^{-x}$.
Statement I $f(c) = \frac{1}{3}$, for some c Ɛ R.
Statement II $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all x Ɛ R.

90. Let A be a 2 x 2 matrix with non-zero entries and $A^2 = I$, where I is 2 x 2 identity matrix. Define Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A.
Statement I Tr(A) = 0
Statement II |A| = 1